

Target Localization: Energy-Information Trade-offs using Mobile Sensor Networks

Wencen Wu¹, Fumin Zhang², and Yorai Wardi²

Abstract—We investigate the energy-information trade-offs in target localization using a group of N sensing agents based on bearing measurements. We aim to design optimal trajectories for the coordinating sensing agents that minimize the kinetic energy and maximize the information collected by the agents along their trajectories. The determinant of Fisher information matrix (FIM) is used as a metric for the information. Inspired by biology, we construct a constrained calculus of variations problem that captures the observation that better information results in more energy dissipation, which requires higher energy supply. We solve the equations of motion as Hamilton equations, which produce a set of ordinary differential equations that can be integrated for the trajectories of the agents. An extended Kalman filter (EKF) is used to produce estimates of the state of the target.

I. INTRODUCTION

Target localization and tracking is an important research topic with various applications such as reconnaissance, surveillance, and search and rescue [1]–[7]. To locate and track a target, mobile sensors take measurements of the target (e.g. bearing, range, time-of-arrival, or signal strength), produce estimates of the state of the target, then, plan their trajectories accordingly. A large body of recent work exists in literature to address related problems such as state estimation [8], trajectory optimization [2], [9] and optimal sensor placement [10]–[12]. One common approach in the existing literature is to formulate an optimization problem given various performance metrics, and apply numerical techniques to determine optimal trajectories of the agents. Receding horizon control or model predictive control are popular numerical methods that solve such optimization problems iteratively when new information about the target is obtained [2], [13], [14].

In biology, the energy-information trade-offs are often observed in routine activities of animal species [15]–[17]. For example, a certain electric fish swim in a less efficient manner that increases the cost of movement in exchange with a higher encounter rate for prey [16]. Research on the trade-offs between speed in making decisions and accuracy in obtaining information in animals such as bees are reviewed in [18]. Inspired by such biological insights, we investigate the

trade-offs between energy and information in target localization using mobile sensor networks. Since the operation time of mobile sensor networks is greatly dependent on the energy supply [19], and the largest amount of energy is consumed by the motion, we focus on the kinetic energy of the mobile sensor networks. As for information, Fisher information matrix (FIM) is a common measure for information quality [2], [9], [11], [12], [20], [21]. Optimal sensor configuration can be derived by maximizing the determinant of FIM [10]–[12]. Therefore, we use the determinant of FIM as a measure of information quality.

We leverage the concept of energy-information trade-offs to plan motion for a mobile sensor network consisting of a group of collaborative mobile sensing agents. We formulate a constrained calculus of variations problem, which is motivated by the design principle that captures the observation that in order to maintain optimal sensor configuration and obtain better information, more energy supply is required to overcome the increased drag force from the environment, leaving less energy for motion (kinetic energy), which is similar to the electric fish case. Since the determinant of FIM only depends on spatial (geometric) variables about the agents and the target, we interpret it as the potential energy of the tracking system. We solve the equations of motion as Hamilton equations [22], which results in a set of ordinary differential equations that can be integrated to provide trajectories of the agents. Our method designs optimal trajectories for the sensing agents that minimize the kinetic energy and maximize the information along their trajectories. The results clearly demonstrate the trade-offs between kinetic energy and information.

We investigated the energy information trade-offs in [23]. In this paper, we extend the results in [23] in several ways. First, we incorporate energy dissipation in the problem formulation, which serves as the constraint for the calculus of variations problem and clearly addresses the energy-information trade-offs. Second, instead of just focusing on the path planning without considering state estimation, in this paper, we construct an extended Kalman filter (EKF) and solve the state estimation and trajectory optimization simultaneously, which is more realistic in practice. Third, we extend the analysis from two agents to N -agent groups. The advantages of considering N -agent groups include increased information collected simultaneously and adaptiveness to the environment and possible system failure.

Our work differs from existing literature of target localization in that we formulate a constrained calculus of variations problem, which takes into account the energy-information

*The research work is supported by ONR grants N00014-09-1-1074 and N00014-10-10712 (YIP), and NSF grants ECCS-0841195 (CAREER), CNS-0931576, OCE-1032285, IIS-1319874, and CNS-1239225.

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trade-offs that are often observed in animal species. Under our framework, direct integration of the Hamilton equations every time a new state estimation is obtained provides trajectories of the agents, which is more computational efficient than existing approaches based on path planning or model predictive control that solve a fresh optimization problem every time.

The rest of the paper is organized as follows. Section II introduces background about bearings-only target tracking and the Fisher Information and discusses the state estimation using EKF. Section III introduces the problem formulation of target localization using a mobile sensor network with the consideration of energy-information trade-offs and provides solutions for trajectories of the agents. Section IV illustrates the simulation results, and Section V concludes the paper.

II. BACKGROUND: BEARINGS-ONLY TARGET LOCALIZATION

In this section, we set up the problem as N agents localizing one target based on bearing-only measurements. We then derive the determinant of FIM that quantifies the information collected by the N agents and construct an EKF that produces estimates of the state of the target.

A. Tracking Model

As illustrated in Fig. 1, let (x, y) be the inertial frame and \mathbf{r}_T and \mathbf{r}_i be positions of a target and the i th agent in the frame, $i = 1 \dots N$, respectively. The agent takes bearing measurements of the target $\theta_i \in [-\pi, \pi]$ with respect to the x axis. d_i is the distance from the i th agent to the target that satisfies $d_i = \|\mathbf{r}_i - \mathbf{r}_T\|$. Let $\mathbf{r}_T = (r_{T,x}, r_{T,y})$ be the state of the target and let $\mathbf{r}_i = (r_{i,x}, r_{i,y})$. Then, from the geometry shown in Fig. 1, the bearing angle θ_i satisfies

$$\theta_i = \arctan \frac{r_{T,y} - r_{i,y}}{r_{T,x} - r_{i,x}}. \quad (1)$$

We assume that each agent is aware of its position \mathbf{r}_i in the frame and shares its measurement with a central controller. At each time instant, after collecting the measurements from the N agents, the central controller produces an estimate of the state of the target $\hat{\mathbf{r}}_T$. Under the assumption that each agent only takes one measurement at each time instant, at least two agents are required to locate a target. The goal is to design trajectories for the N agents based on the estimated state of the target $\hat{\mathbf{r}}_T$ so that the kinetic energy and the information collected by the N agents along their trajectories demonstrate trade-offs that mimic behaviors of certain animal species.

B. Extended Kalman Filter

Let y_i be the measurement obtained by the i th agent at time t and w be the measurement noise. Then, we have

$$y_i = \theta_i + w = \arctan \frac{r_{T,y} - r_{i,y}}{r_{T,x} - r_{i,x}} + w. \quad (2)$$

Denote \mathbf{y} as the vector consisting of all the measurements collected from the N collaborating agents. That is, $\mathbf{y} = [y_1 \dots, y_N]^T$. Define $h(\mathbf{r}_T) = [\theta_1, \dots, \theta_N] =$

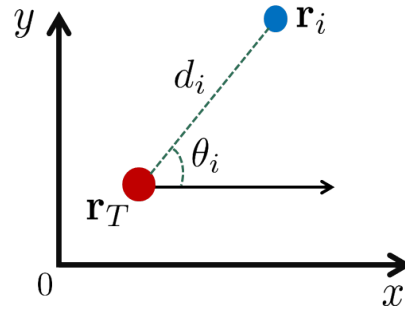


Fig. 1. Geometry of the tracking model.

$[\arctan \frac{r_{T,y} - r_{1,y}}{r_{T,x} - r_{1,x}}, \dots, \arctan \frac{r_{T,y} - r_{N,y}}{r_{T,x} - r_{N,x}}]^T$, then, the measurement equation for the N sensing agents is

$$\mathbf{y} = h(\mathbf{r}_T) + \mathbf{w}, \quad (3)$$

where $\mathbf{w} \sim \mathcal{N}(0, R)$ is zero-mean Gaussian noise vector with covariance matrix $R = \sigma_w^2 I$, in which I is an identity matrix.

Suppose the state equation is given by

$$\dot{\mathbf{r}}_T = f(\mathbf{r}_T) + \varepsilon, \quad (4)$$

where $f(\mathbf{r}_T)$ is the state transition model and ε is assumed to be zero-mean Gaussian noise with covariance Q . We construct an EKF to produce the state estimation, since the measurement equation (3) is nonlinear. Other filtering algorithms such as least-squares estimation [8], maximum likelihood estimation [24], and Pseudolinear filtering [25] can also be utilized without affecting the main results.

Define \mathbf{H} as the Jacobian of the measurement vector with respect to the state of the target. We derive $\mathbf{H} = \nabla_{\mathbf{r}_T} h(\mathbf{r}_T) = \left(\frac{\partial \theta_1}{\partial \mathbf{r}_T} \dots \frac{\partial \theta_N}{\partial \mathbf{r}_T} \right)^T$, the i th row of which is

$$\mathbf{H}_i = \left(-\frac{r_{T,y} - r_{i,y}}{(r_{T,x} - r_{i,x})^2 + (r_{T,y} - r_{i,y})^2} \quad \frac{r_{T,x} - r_{i,x}}{(r_{T,x} - r_{i,x})^2 + (r_{T,y} - r_{i,y})^2} \right). \quad (5)$$

Given state equation (4) and measurement equation (3), the EKF can be constructed following standard steps:

$$\begin{aligned} \hat{\mathbf{r}}_T &= f(\mathbf{r}_T) + K(\mathbf{y} - h(\mathbf{r}_T)), \\ \dot{P} &= FP + PF^T - K\hat{\mathbf{H}}P + Q, \\ K &= P\hat{\mathbf{H}}^T R^{-1}, \end{aligned} \quad (6)$$

where $F = \frac{\partial f}{\partial \mathbf{r}_T} |_{\hat{\mathbf{r}}_T}$ and $\hat{\mathbf{H}} = \mathbf{H} |_{\hat{\mathbf{r}}_T}$. At each time instant, the updated state estimation from the EKF is used in the trajectory planning for the agents, which will be discussed in Section III-C.

C. Fisher Information

A common measure of the performance of an estimator is the error variance, which indicates the uncertainty associated with the estimates obtained by the estimator. The Cramer-Rao lower bound (CRLB) provides a lower bound of the variance of an estimator given a set of measurements. The inverse of the Cramer-Rao lower bound is referred to as the Fisher information, which describes the amount of information that the measurement \mathbf{y} carries about the state.

As stated in [10]–[12], [21], a configuration of the agents over the space of all angle positions $\theta_i, \forall i \in \{1, \dots, N\}$ is optimal if the configuration maximizes the determinant of Fisher information matrix, which is typically referred as the D-optimality criterion.

Following similar derivations in [10]–[12], [21], we obtain the FIM of the tracking system given the measurement equation (3) as

$$\begin{aligned} \mathbf{I}(\mathbf{r}_T) &= \nabla_{\mathbf{r}_T} h(\mathbf{r}_T)^T R^{-1} \nabla_{\mathbf{r}_T} h(\mathbf{r}_T) \\ &= \frac{1}{\sigma_w^2} \begin{pmatrix} \sum_{i=1}^N \frac{1}{d_i^2} \sin^2 \theta_i & -\sum_{i=1}^N \frac{1}{2d_i^2} \sin 2\theta_i \\ -\sum_{i=1}^N \frac{1}{2d_i^2} \sin 2\theta_i & \sum_{i=1}^N \frac{1}{d_i^2} \cos^2 \theta_i \end{pmatrix}. \end{aligned} \quad (7)$$

The determinant of the FIM can be calculated as

$$\det(\mathbf{I}) = \frac{1}{\sigma_w^2} \sum_S \frac{\sin^2(\theta_i - \theta_j)}{d_i^2 d_j^2}, \quad (8)$$

where S is the set that contains all pairs of i and j , $i, j = 1, \dots, N, j > i$. The determinant of FIM implies that to obtain maximum information when tracking a target, the neighboring agents should triangulate, i.e., $\theta_i - \theta_j \rightarrow \frac{\pi}{2}$ and move towards the target, i.e., $d_i \rightarrow 0$.

III. ENERGY-INFORMATION TRADE-OFFS

In this section, we formulate the problem of target localization using mobile sensor networks with the consideration of energy-information trade-offs. By connecting the problem with Hamiltonian mechanics, we obtain solutions for the trajectories of the agents as Hamilton equations.

A. Energy-Information Trade-offs in Motion and Sensing

In mobile sensor networks, motion, communication, and sensor information processing all consume energy. However, motion usually is the largest user of energy for mobile agents. Therefore, we focus on the kinetic energy of the tracking system. Suppose the agents are point masses satisfying $\dot{\mathbf{r}}_i = \mathbf{v}_i, i = 1, \dots, N$, in which $\mathbf{v}_i, i = 1, \dots, N$ are the velocities of the agents. The kinetic energy of the N -agent system is

$$E = \frac{1}{2} \sum_{i=1}^N \|\mathbf{v}_i\|^2. \quad (9)$$

To represent the information quality, we define

$$V = \frac{K_1}{\sigma_w^2} \sum_S \frac{\sin^2(\theta_i - \theta_j)}{d_i^2 d_j^2}, j > i, \quad (10)$$

where K_1 is a constant. V is proportional to the determinant of FIM.

We aim to find optimal trajectories for the agents so that the kinetic energy is minimized while the information is maximized. For this purpose, we define a cost function as

$$J = \int_0^T (E - V) dt, \quad (11)$$

where T is the terminal time. Similar to the traditional potential energy, V only depends on spatial displacements of the agents relative to the target. Therefore, we interpret it

as the potential energy of the tracking system. Hamiltonian mechanics tells us that if we consider the agents as Newtonian particles only subjecting to potential forces introduced by potential energy V without dissipation, then, the total energy of the system is conserved along the trajectories of the N agents [22]. In other words, the Hamiltonian H , which represents the total energy of the system: $H = E + V$, remains constant along the trajectories of the agents. However, dissipation may exist due to water or wind resistances, or other kinds of friction forces. Therefore, extra energy supplies are needed to maintain a constant value for H .

We draw inspirations from electric fish swimming with their body tilted while moving through space searching for prey. By swimming in this posture, the body area that contacts with water increases to increase the amount of information collected from the environment, resulting in increased drags from water resistance, which further results in increased energy consumption due to friction. If we consider the cost of movement as energy provided internally by the fish, which usually has an upper limit, then, with the increased energy dissipation due to friction, less energy is left for motion. In other words, the improvement of sensory performance, which leads to higher prey encounter rate, is associated with decreased energy supply for motion [16]. If we interpret the prey encounter rate as information collected by the sensors on the body of the fish, then, the observation leads to a design principle that more information results in more energy dissipation due to the friction, leaving less energy for motion, which is referred to as the energy-information trade-off principle.

In a mobile sensor network, each agent represents a sensor node. As we discussed in Section II-C, to obtain better information, the agents are required to maintain a desired formation, the topology of which determines the sensing area the mobile sensor network covers. This may lead to different energy dissipation due to the drags and frictions from the environment. Therefore, for the target tracking system moving in an environment with drag forces, we model the drag coefficient as $K = K_2 \det(\mathbf{I})$, in which K_2 is a constant. K implies that better information is associated with larger drag force.

For path planning, define \mathbf{v} as the velocity vector consisting of the velocities of all the agents $\mathbf{v}_i, i = 1 \dots, N$. $\mathbf{v} = [\mathbf{v}_1, \dots, \mathbf{v}_N]^T$. Given the drag coefficient, the drag force applied to the N -agent tracking system is calculated as $\mathbf{F} = K\mathbf{v}$. Note that the drag force can be modeled as higher order functions of \mathbf{v} , which can be handled by our design too. In this paper, we consider the case $\mathbf{F} = K\mathbf{v}$ for simplicity. The energy dissipation caused by the drag force is then given by

$$\mathbf{F} \cdot \mathbf{v} = K_2 \det(\mathbf{I}) \|\mathbf{v}\|^2 = \frac{2K_2}{K_1} EV, \quad (12)$$

which implies that higher information and higher speed result in more energy dissipation. In most cases, a dynamic system only has a limited energy supply. Let E_s be the maximum

energy supply for the tracking system, then, we require

$$\frac{2K_2}{K_1}EV \leq E_s. \quad (13)$$

With the above discussion, we formulate a constrained calculus of variations problem with fixed terminal time and free terminal state with cost function (11) and constraint (13). The problem is as follows.

Problem 3.1: With the updated state estimation $\hat{\mathbf{r}}_T$ at every time instant, find optimal trajectories $\hat{\mathbf{r}}_i = \mathbf{v}_i$ for agents $1, \dots, N$, such that the cost function (11) is minimized subjecting to the constraint (13).

B. Optimal Trajectories

In this section, we provide solutions to the constrained calculus of variations problem in Problem 3.1. To unify the variables in both kinetic and potential energy, we perform coordinate transform from \mathbf{r}_i to d_i and θ_i using

$$\mathbf{r}_{i,x} = d_i \cos \theta_i, \quad \text{and} \quad \mathbf{r}_{i,y} = d_i \sin \theta_i. \quad (14)$$

If we consider \mathbf{v}_i as the relative velocity from the i th agent to the target and v_i as the speed along the trajectory, then, we derive $v_i^2 = \dot{d}_i^2 + d_i^2 \dot{\theta}_i^2$. The kinetic energy of the system (9) becomes $E = \frac{1}{2} \sum_{i=1}^N (\dot{d}_i^2 + d_i^2 \dot{\theta}_i^2)$.

Under this framework, we define generalized coordinates \mathbf{q} and generalized velocities $\dot{\mathbf{q}}$ for the relative distances and bearing angles from the agents to the target. Let $\mathbf{q} = [q_1^d, \dots, q_N^d, q_1^\theta, \dots, q_N^\theta]$, in which $q_i^d = d_i$ and $q_i^\theta = \theta_i, i = 1, \dots, N$. Then, the kinetic energy and potential energy can be rewritten as

$$E(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2} \sum_{i=1}^N ((\dot{q}_i^d)^2 + (q_i^d \dot{q}_i^\theta)^2),$$

$$V(\mathbf{q}) = \frac{K_1}{\sigma_w^2} \sum_S \frac{\sin^2(q_i^\theta - q_j^\theta)}{(q_i^d q_j^d)^2}. \quad (15)$$

Thus, the cost function becomes

$$J(\mathbf{q}, \dot{\mathbf{q}}) = \int_0^T (E(\mathbf{q}, \dot{\mathbf{q}}) - V(\mathbf{q})) dt$$

$$= \int_0^T \frac{1}{2} \sum_{i=1}^N ((\dot{q}_i^d)^2 + (q_i^d \dot{q}_i^\theta)^2) - \frac{K_1}{\sigma_w^2} \sum_S \frac{\sin^2(q_i^\theta - q_j^\theta)}{(q_i^d q_j^d)^2} dt$$

$$= \int_0^T L(\mathbf{q}, \dot{\mathbf{q}}) dt, \quad (16)$$

where $L(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2} \sum_{i=1}^N ((\dot{q}_i^d)^2 + (q_i^d \dot{q}_i^\theta)^2) - \frac{K_1}{\sigma_w^2} \sum_S \frac{\sin^2(q_i^\theta - q_j^\theta)}{(q_i^d q_j^d)^2}$ is the Lagrangian of the system. The cost function is actually called the *action* in Lagrangian mechanics. Hamilton's principle of least action states that the trajectories of the system is a stationary solution of the action [22].

With the presence of energy dissipation, we denote the constraint as

$$m(\mathbf{q}, \dot{\mathbf{q}}) = \frac{2K_2}{K_1} E(\mathbf{q}, \dot{\mathbf{q}}) V(\mathbf{q}) - E_s \leq 0. \quad (17)$$

Define $\frac{2K_2}{K_1} E(\mathbf{q}, \dot{\mathbf{q}}) V(\mathbf{q})$ as E_d and refer to it as the dissipated energy. The problem is to find \mathbf{q} and $\dot{\mathbf{q}}$ such that the

cost function (16) is minimized subjecting to the constraint (17). We consider the problem as a constrained calculus of variations problem with fixed terminal time and free terminal state.

To solve this problem, we define an augmented Lagrangian

$$L_a(\mathbf{q}, \dot{\mathbf{q}}) = E(\mathbf{q}, \dot{\mathbf{q}}) - V(\mathbf{q}) + \lambda m(\mathbf{q}, \dot{\mathbf{q}})$$

$$= E(\mathbf{q}, \dot{\mathbf{q}}) - V(\mathbf{q}) + \frac{2\lambda K_2}{K_1} E(\mathbf{q}, \dot{\mathbf{q}}) V(\mathbf{q}) - \lambda E_s, \quad (18)$$

in which λ is a Lagrange multiplier. $\lambda = 0$ corresponds to the case when the energy dissipation E_d is less than the energy supply E_s . When the constraint is active, $\lambda > 0$ according to KKT conditions [26].

The Hamiltonian of the system translates n second-order Lagrangian equations into $2n$ first-order equations, where n is the number of degrees of freedom of the system [22]. Define the generalized momenta as $\mathbf{p} = \frac{\partial L}{\partial \dot{\mathbf{q}}}$. The Hamiltonian and the Lagrangian are related by the Legendre transformation $\dot{\mathbf{q}} = h(\mathbf{q}, \mathbf{p}, t)$ as follows: $H(\mathbf{q}, \mathbf{p}, t) = \mathbf{p}^T h(\mathbf{q}, \mathbf{p}, t) - L(\mathbf{q}, h(\mathbf{q}, \mathbf{p}, t), t)$. Define $\mathbf{p} = [p_1^d, \dots, p_N^d, p_1^\theta, \dots, p_N^\theta]$. We calculate that

$$p_i^d = \frac{\partial L_a(\mathbf{q}, \dot{\mathbf{q}})}{\partial \dot{q}_i^d} = \frac{\partial E(\mathbf{q}, \dot{\mathbf{q}})}{\partial \dot{q}_i^d} \left(1 + \frac{2\lambda K_2}{K_1} V(\mathbf{q})\right)$$

$$= \dot{q}_i^d \left(1 + \frac{2\lambda K_2}{K_1} V(\mathbf{q})\right),$$

$$p_i^\theta = \frac{\partial L_a(\mathbf{q}, \dot{\mathbf{q}})}{\partial \dot{q}_i^\theta} = \frac{\partial E(\mathbf{q}, \dot{\mathbf{q}})}{\partial \dot{q}_i^\theta} \left(1 + \frac{2\lambda K_2}{K_1} V(\mathbf{q})\right)$$

$$= \dot{q}_i^\theta (q_i^d)^2 \left(1 + \frac{2\lambda K_2}{K_1} V(\mathbf{q})\right), \quad (19)$$

which lead to the Legendre transformation

$$\dot{q}_i^d = \frac{K_1 p_i^d}{K_1 + 2\lambda K_2 V(\mathbf{q})}, \quad \text{and} \quad \dot{q}_i^\theta = \frac{K_1 p_i^\theta}{(q_i^d)^2 (K_1 + 2\lambda K_2 V(\mathbf{q}))}. \quad (20)$$

Substituting \dot{q}_i^d and \dot{q}_i^θ into $E(\mathbf{q}, \dot{\mathbf{q}})$ and defining $E(\mathbf{q}, \mathbf{p}) = \frac{1}{2} \sum_{i=1}^N \left((p_i^d)^2 + \frac{(p_i^\theta)^2}{(q_i^d)^2} \right)$, we obtain $E(\mathbf{q}, \dot{\mathbf{q}}) = \frac{K_1^2}{(K_1 + 2\lambda K_2 V(\mathbf{q}))^2} E(\mathbf{q}, \mathbf{p})$. We calculate the Hamiltonian of the system as

$$H(\mathbf{q}, \mathbf{p}) = \mathbf{p}^T h(\mathbf{q}, \mathbf{p}) - L(\mathbf{q}, h(\mathbf{q}, \mathbf{p}))$$

$$= \frac{2K_1}{(K_1 + 2\lambda K_2 V(\mathbf{q}))} E(\mathbf{q}, \mathbf{p}) - \frac{K_1^2}{(K_1 + 2\lambda K_2 V(\mathbf{q}))^2} E(\mathbf{q}, \mathbf{p})$$

$$+ V(\mathbf{q}) - \frac{2\lambda K_2}{K_1} \frac{K_1^2}{(K_1 + 2\lambda K_2 V(\mathbf{q}))^2} E(\mathbf{q}, \mathbf{p}) V(\mathbf{q}) + \lambda E_s$$

$$= \frac{K_1}{(K_1 + 2\lambda K_2 V(\mathbf{q}))} E(\mathbf{q}, \mathbf{p}) + V(\mathbf{q}) + \lambda E_s, \quad (21)$$

from which we derive the following equations by applying the Hamilton equations $\dot{\mathbf{q}} = \frac{\partial H}{\partial \mathbf{p}}$ and $\dot{\mathbf{p}} = -\frac{\partial H}{\partial \mathbf{q}}$:

$$\dot{\mathbf{q}} = \frac{\partial H}{\partial \mathbf{p}} = \frac{K_1}{(K_1 + 2\lambda K_2 V(\mathbf{q}))} \frac{\partial E(\mathbf{q}, \mathbf{p})}{\partial \mathbf{p}}, \quad (22)$$

and

$$\begin{aligned} \dot{\mathbf{p}} = & -\frac{\partial H}{\partial \mathbf{q}} = -\frac{K_1}{(K_1 + 2\lambda K_2 V(\mathbf{q}))} \frac{\partial E(\mathbf{q}, \mathbf{p})}{\partial \mathbf{q}} \\ & + \frac{2\lambda K_1 K_2 E(\mathbf{q}, \mathbf{p})}{(K_1 + 2\lambda K_2 V(\mathbf{q}))^2} \frac{\partial V(\mathbf{q})}{\partial \mathbf{q}} - \frac{\partial V(\mathbf{q})}{\partial \mathbf{q}}. \end{aligned} \quad (23)$$

$\dot{\mathbf{q}}$ is known as in Equation (20). We calculate that

$$\begin{aligned} \frac{\partial E(\mathbf{q}, \mathbf{p})}{\partial q_i^d} &= \frac{(p_i^\theta)^2}{(q_i^d)^3}, \\ \frac{\partial E(\mathbf{q}, \mathbf{p})}{\partial q_i^\theta} &= 0, \\ \frac{\partial V(\mathbf{q})}{\partial q_i^d} &= -\frac{2K_1}{\sigma_w^2} \left(\sum_{j=1, \dots, N, j \neq i} \frac{\sin^2(q_j^\theta - q_i^\theta)}{(q_i^d)^3 (q_j^d)^2} \right), \\ \frac{\partial V(\mathbf{q})}{\partial q_i^\theta} &= \frac{K_1}{\sigma_w^2} \sum_{j=1, \dots, N, j \neq i} \frac{\sin(2(q_i^\theta - q_j^\theta))}{(q_i^d q_j^d)^2}. \end{aligned} \quad (24)$$

In addition, λ is determined by the constraint

$$m(\mathbf{q}, \mathbf{p}) = \frac{2K_1 K_2}{(K_1 + 2\lambda K_2 V(\mathbf{q}))^2} E(\mathbf{q}, \mathbf{p}) V(\mathbf{q}) - E_s = 0. \quad (25)$$

The trajectories of the agents are obtained by plugging Equation (24) into Equations (23) and (22) and integrating Equations (23) and (22).

C. The Target Localization Algorithm

We now summarize the procedure of localizing a stationary target using N mobile sensing agents with the presence of dissipation as follows.

Algorithm 3.2: Consider N sensing agents with bearing sensors and a central controller that receives information from all the agents. At $t = 0$, the initial locations of the agents are $[\mathbf{r}_1(0), \dots, \mathbf{r}_N(0)]$. Repeat:

- S.1 At time step t , each agent takes a bearing measurement $y_i(t)$ of the target as shown in Equation (2).
- S.2 the central controller runs the EKF that produces an estimate of the location of the target $\hat{\mathbf{r}}_T$ according to Equation (6), and computes $\hat{d}_i(t)$, $\hat{d}_i'(t)$, and $\hat{\theta}_i(t)$.
- S.3 the central controller performs coordinate transform to obtain $\hat{\mathbf{q}}(t)$ and $\hat{\mathbf{p}}(t)$, and calculate $\mathbf{q}(t+1)$ and $\mathbf{p}(t+1)$ by integrating Equations (23) and (22) after solving Equation (25).
- S.4 the central controller determine the next locations of the agents $\mathbf{r}_i(t+1)$ using Equation (14).
- S.5 the agents move to new locations $\mathbf{r}_i(t+1)$.

The algorithm stops when $t = T$.

IV. SIMULATION RESULTS

In this section, we demonstrate simulation results of N agents localizing one stationary target that sits at $(0,0)$. We simulate two cases: $N = 2$ and $N = 4$. Fig. 2 and Fig. 3 illustrate the trajectories of $N = 2$ and $N = 4$ agents, respectively. The initial locations of the agents are at $(1, 1.4)$ and $(1, -1.4)$ in Fig. 2, and are at $(-2.5, 2.7)$, $(2.3, 1.8)$, $(-1.9, 2.4)$ and $(2.2, -2.7)$ in Fig. 3. The initial locations are indicated by the stars in the two figures. The dots are

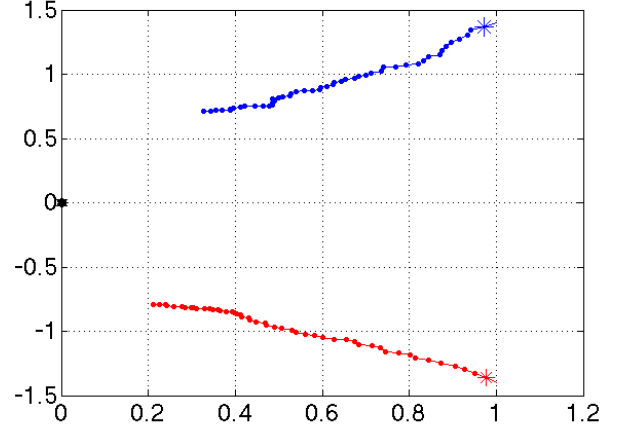


Fig. 2. Trajectories of two agents localizing one stationary target with drag forces from the environment.

the positions of the agents at each time instant. The initial velocity of all the agents is $[-2, 0]^T$.

Following Algorithm 3.2, at each time step, each agent takes one noisy bearing measurement of the target. Then a central controller collects all the measurements, runs the extended Kalman filter to produce the state estimation of the target, and calculates the next positions of the agents by solving Equations (22) and (23).

Since the dots in the figures represent the positions of the agents at each time instant and the time step is constant in the simulation, we can observe from Figs. 2 and Fig. 3 that, while moving towards the target, the agents reduce their speed to reduce their kinetic energy (the distances between two adjunct dots are getting closer). In the meanwhile, the differences of the bearing angles between two adjunct agents are kept close to $\frac{\pi}{2}$, which aims to maximize the information. Fig. 4 demonstrates the kinetic energy E (red dotted line), the information V (blue solid line), the cost function L (green dashed line), and the Lagrange multiplier λ (magenta dash-dot line) of the four-agent tracking system. In this case, the maximum energy supply to the tracking system is $E_s = 0.6$. As we can clearly observe from Fig. 4, the constraint (17) is inactive along the trajectories for most of the time ($\lambda = 0$), indicating that the maximum amount of energy dissipation is less than the energy supply E_s . When the energy dissipation of the system exceeds the energy supply, for example, the agents speed up so that the kinetic energy increases, the constraint becomes active ($\lambda > 0$), which regulates the speed of the agents. We can also observe from Fig. 4 that along the trajectories of the agents, the kinetic energy E keeps reducing and information V keeps increasing, demonstrating the energy-information trade-offs in the tracking system. It is worth mentioning that with the increased energy supply E_s , we can obtain better information (V increases). This is consistent with the observation from electric fish that better information requires more energy supply.

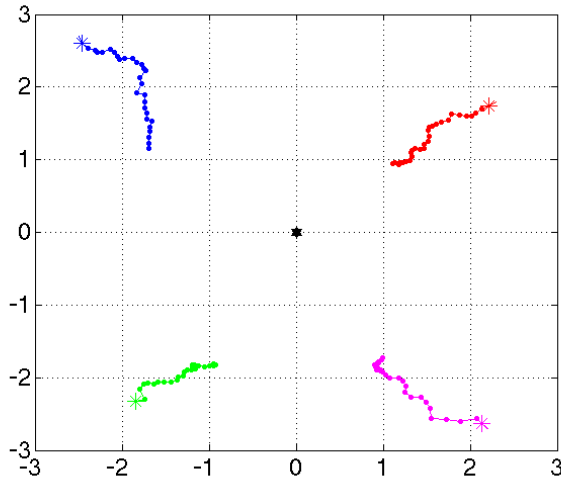


Fig. 3. Trajectories of four agents localizing one stationary target with drag forces from the environment.

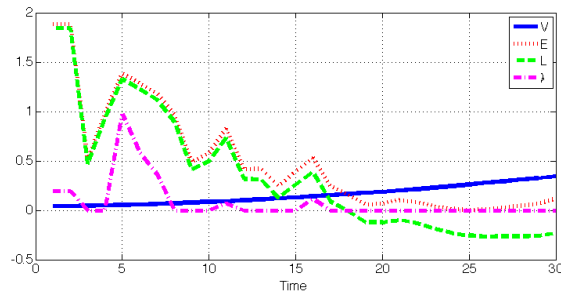


Fig. 4. Information, kinetic energy, cost function, and Lagrange multiplier along trajectories of four agents localizing one stationary target with drag forces from the environment.

V. CONCLUSIONS AND FUTURE WORK

In this paper, we investigate the energy-information trade-offs in target localization using N sensing agents based on bearing measurements. Inspired by observations in biology, we design optimal trajectories for a group of coordinating sensing agents that minimize the kinetic energy and maximize the information collected by the agents along their trajectories based on a design principle that more information requires more energy supply. The determinant of FIM is used as a metric for the information. An EKF is used to produce state estimation of the target. We construct a constrained calculus of variations problem and solve the equations of motion using Hamilton equations, which produces a set of ODEs that can be solved for the trajectories of the agents. Future work includes extension to distributed algorithms and tracking of moving targets.

REFERENCES

- [1] B. Grocholsky, J. Keller, V. Kumar, and G. Pappas, "Cooperative air and ground surveillance," *IEEE Robotics and Automation Magazine*, vol. 13, no. 3, pp. 16–25, 2006.
- [2] J. Ousingawat and M. E. Campbell, "Optimal cooperative reconnaissance using multiple vehicles," *Journal of Guidance, Control, and Dynamics*, vol. 30, no. 1, pp. 122–135, 2007.

- [3] Z. Tang and U. Ozguner, "Motion planning for multitarget surveillance with mobile sensor agents," *IEEE Transactions on Robotics*, vol. 21, no. 5, pp. 898–908, 2005.
- [4] M. Wheeler, B. Schrick, W. Whitacre, M. Campbell, R. Rysdyk, and R. Wise, "Cooperative tracking of moving targets by a team of autonomous UAVs," in *Proc. IEEE/AIAA 25th Digital Avionics Systems Conf*, 2006, pp. 1–9.
- [5] B. Bethke, M. Valenti, and J. How, *Advances in Cooperative Control and Optimization*, 2007, vol. 369, ch. Cooperative Vision Based Estimation and Tracking Using Multiple UAVs, pp. 179–189.
- [6] M. E. Campbell and W. W. Whitacre, "Cooperative tracking using vision measurements on seascan UAVs," *IEEE Transactions on Control Systems Technology*, vol. 15, no. 4, pp. 613–626, 2007.
- [7] S. Oh, L. Schenato, P. Chen, and S. Sastry, "Tracking and coordination of multiple agents using sensor networks: System design, algorithms and experiments," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 234–254, 2007.
- [8] K. Dogancay, "Bearings only target localization using total least squares," *Signal Processing*, vol. 85, no. 9, pp. 1695–1710, 2003.
- [9] S. S. Ponda, R. M. Kolacinski, and E. Frazzoli, "Trajectory optimization for target localization using small unmanned aerial vehicles," in *In AIAA Guidance, Navigation, and Control Conference*, 2009.
- [10] S. MartıNez and F. Bullo, "Optimal sensor placement and motion coordination for target tracking," *Automatica*, vol. 42, no. 4, pp. 661–668, 2006.
- [11] A. N. Bishop, B. Fidan, B. Anderson, K. Doğançay, and P. N. Pathirana, "Optimality analysis of sensor-target localization geometries," *Automatica*, vol. 46, no. 3, pp. 479–492, 2010.
- [12] S. Zhao, B. M. Chen, and T. H. Lee, "Optimal placement of bearing-only sensors for target localization," in *Proc. of 2012 American Control Conference*, 2012, pp. 5108–5113.
- [13] E. W. Frew, J. Langelaan, and M. Stachura, "Adaptive planning horizon based on information velocity for vision-based navigation," in *AIAA Guidance, Navigation and Controls Conference*, 2007.
- [14] K. Yang, S. Sukkarieh, and Y. Kang, "Adaptive nonlinear model predictive path tracking control for a fixed-wing unmanned aerial vehicle," in *AIAA Guidance, Navigation & Control Conference*, 2009.
- [15] P. Skorupski, J. Spaethe, and L. Chittka, "Visual search and decision making in bees: Time, speed, and accuracy," *International Journal of Comparative Psychology*, vol. 19, pp. 342–357, 2006.
- [16] M. A. MacIver, N. A. Patankar, and A. A. Shirgaonkar, "Energy-information trade-offs between movement and sensing," *PLoS Comput Biol*, vol. 6, no. 5, p. e1000769, 2010.
- [17] S. A. Stamper, E. Roth, N. J. Cowan, and E. S. Fortune, "Active sensing via movement shapes spatiotemporal patterns of sensory feedback," *The Journal of Experimental Biology*, vol. 215, pp. 1567–1574, 2012.
- [18] L. Chittka and P. S. ann Nigel E. Raine, "Speed-accuracy tradeoffs in animal decision making," *Trends in Ecology and Evolution*, vol. 24, no. 7, pp. 400–407, 2009.
- [19] F. Zhang, Z. Shi, and S. Mukhopadhyay, "Robustness analysis for battery-supported cyber-physical systems," *ACM Transactions on Embedded Computing Systems (TECS)*, vol. 12, no. 3, p. 69, 2013.
- [20] Y. Oshman and P. Davidson, "Optimization of observer trajectories for bearings-only target localization," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 35, no. 3, pp. 892–902, 1999.
- [21] K. Doğançay and H. Hmam., "Optimal angular sensor separation for AOA localization," *Signal Processing*, vol. 88, no. 5, pp. 1248–1260, 2008.
- [22] L. N. Hand and J. D. Finch, *Analytical Mechanics*. Cambridge University Press, 1998.
- [23] W. Wu, F. Zhang, and Y. Wardi, "Energy-information tradeoffs in motion and sensing for target localization," in *Proc. of 2013 European Control Conference*, 2013, pp. 1250–1255.
- [24] J. H. de Vlieger and R. H. J. G. Meyling, "Maximum likelihood estimation for long-range target tracking using passive sonar measurements," *IEEE Transactions on Signal Processing*, vol. 40, no. 5, pp. 1216–1225, 1992.
- [25] V. Aidala and S. Hammel, "Utilization of modified polar coordinates for bearings-only tracking," *IEEE Transactions on Automatic Control*, vol. 28, no. 3, pp. 283–294, 1983.
- [26] M. Avriel, *Nonlinear programming: analysis and methods*. Courier Dover Publications, 2012.