Nonlinear Observer Design for Current Estimation based on Underwater Vehicle Dynamic Model

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***Abstract*—As applications for autonomous ocean vehicles expand into more dynamic and constrained environments, such as shallow, coastal areas, the benefits of using more precise dynamic model for control and estimation become more compelling. This paper presents a nonlinear observer for current estimation based on AUV dynamic model. Here, AUV dynamic model in currents is taken into consideration. Motivated by the design method of high-gain observer, we take the current disturbances as the uncertainties of the vehicle dynamic system and design the observer gain matrix with the goal of making the observer robust to the effect of current disturbances. The nonlinear observer estimates vehicle's relative velocity firstly; current velocity is further calculated in an indirect way. The proposed current estimation method is validated by numerical simulation.**

***Index Terms*—Underwater vehicle, dynamic model, nonlinear observer, current estimation**

I. Introduction

Autonomous underwater vehicles (AUVs) are playing an increasingly important role in ocean observation. Although underwater vehicles often operate in time-varying, nonuniform currents, the effect of current on vehicle dynamics is typically ignored in motion models used for control and estimation. Ref.[1] addresses the problem of dynamic positioning and way-point tracking of AUVs in the presence of unknown ocean currents. An exponential observer for the current is designed, aiming to provide current disturbance information for control compensation. In their case, the observer is derived at the kinematic level, while the current is considered as constant. Ref.[2] presents a controller utilizing an estimation of current velocity to compensate environmental disturbances. A nonlinear Luenberger observer is designed with the assumptions that the vehicle velocity, position and attitude are all measured. Ref.[3] introduces modified linear terminal guidance algorithm with an ocean current observer for AUV docking. The method estimates the effect of ocean currents and reduces the vehicle's drift. In their study, a current observer is designed based on AUV kinematic model and the current is actually taken as constant.

As applications for autonomous ocean vehicles expand into more dynamic and constrained environments, such as shallow, coastal areas, the benefits of using more precise dynamic model for control and estimation become more compelling. To incorporate unsteady, nonuniform flow effects, the flow field can be assumed to comprise a steady, nonuniform component and an unsteady, uniform component [4]. Under this assumption, the "full" dynamic model of AUV in currents can be derived [5]. With an accurate dynamic model for an underwater vehicle in currents, one might identify more flow characteristics by observer design. The current information might then be used to enhance navigation precision and control performance or to provide additional data for ocean scientists.

Based on glider's full dynamics in currents, a linear observer has been developed in [6] to estimate flow gradient during glider's zigzag motion in the vertical plane, which uses much fewer measurements, such as depth, pitch angle and moving mass position. Since the nonlinear dynamics of the vehicle has been lost in the linear observer, there could exist a discrepancy between the true and the estimated flow gradients, which increases during motion transitions between equilibrium gliding conditions. Therefore, it is necessary to move forward and carry out nonlinear observer study based on the full dynamic model, not only to increase the accuracy of the estimated results, but also to expand the diversity of the flow characteristics. After all, flow velocity is more interesting and useful, comparing to flow gradient.

In this paper, based on AUV dynamic model in currents, a nonlinear observer is developed in order to estimate the current velocity, just using the measurements of vehicle's absolute velocity and attitude. High-gain observer is a robust tool for observer design and well-used in nonlinear feedback control, which has been fully studied for several decades [7]. For high-gain observer, all the disturbances are considered as the uncertainties of the system which can be estimated as the states of the system. Motivated by the design method of high-gain observer, here we take the current disturbances as the uncertainties of the vehicle dynamic system and estimate flow velocity in an indirect way. The paper is organized as follows. Section II briefly presents the rigid body dynamic model for AUV operating in an unsteady, nonuniform flow field, based on which, observer design and numerical simulation can be carried on. In Section III, the nonlinear observer is designed for current velocity estimation; moreover, the performance of the observer is validated by numerical simulation. Section IV summarizes the main contributions and describes some additional avenues for continuing research.

II. Dynamic Model In Nonuniform Currents

In this section, we present the dynamic equations for the streamlined AUV operating in an unsteady, nonuniform flow, which will be further used for nonlinear observer design and numerical simulation in Section III.

A. Kinematics

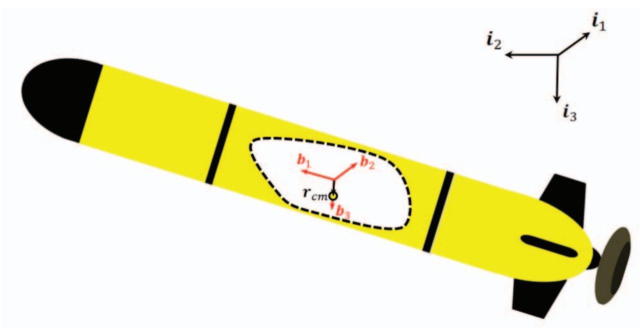


Fig. 1. An underwater vehicle model.

Let $\boldsymbol{X} = [\xi,\ \eta,\ \zeta]^T$ represent the position vector from the origin of an inertially fixed frame $\{\boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3\}$ to the origin of a body-fixed reference frame $\{\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3\}$; see Figure 1. The vector $\boldsymbol{X}$ is the North-East-Down coordinates, expressed in the inertial frame. The orientation of the body is given by the rotation matrix $\boldsymbol{R}$, which maps free vectors from the body frame to the inertial frame. $\boldsymbol{R}$ is parameterized by Euler angles, that is: the roll angle ϕ, the pitch angle θ and the yaw angle ψ. Let $\boldsymbol{v} = [u,\ v,\ w]^T$ and $\boldsymbol{\omega} = [p,\ q,\ r]^T$ represent the translational and rotational velocities of the body with respect to the inertial frame, but expressed in the body frame, respectively. The kinematic equations are

$$\dot{\boldsymbol{X}} = \boldsymbol{R}\boldsymbol{v} \tag{1}$$

$$\dot{\boldsymbol{R}} = \boldsymbol{R}\hat{\boldsymbol{\omega}} \tag{2}$$

where $\hat{\cdot}$ denotes the 3×3 skew-symmetric matrix satisfying $\hat{\boldsymbol{a}}\boldsymbol{b} = \boldsymbol{a}\times\boldsymbol{b}$ for vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

Following Ref.[4], we consider a flow field $\boldsymbol{V}_{\mathrm{f}}(\boldsymbol{X}, t)$ comprising two components: an unsteady, uniform flow component $\boldsymbol{V}_{\mathrm{u}}(t)$ and a steady, circulating flow component $\boldsymbol{V}_{\mathrm{s}}(\boldsymbol{X})$. These two flow components are more conveniently expressed in the body-fixed reference frame:

$$\boldsymbol{v}_{\mathrm{u}}(\boldsymbol{R}, t) = \boldsymbol{R}^T\boldsymbol{V}_{\mathrm{u}}(t) \quad \text{and} \quad \boldsymbol{v}_{\mathrm{s}}(\boldsymbol{R}, \boldsymbol{X}) = \boldsymbol{R}^T\boldsymbol{V}_{\mathrm{s}}(\boldsymbol{X}).$$

In the moving body frame, the flow field

$$\boldsymbol{v}_{\mathrm{f}}(\boldsymbol{R}, \boldsymbol{X}, t) = \boldsymbol{v}_{\mathrm{u}}(\boldsymbol{R}, t) + \boldsymbol{v}_{\mathrm{s}}(\boldsymbol{R}, \boldsymbol{X}).$$

B. Dynamics

Let $\boldsymbol{\nu}$ represent the generalized velocity of the vehicle and let $\boldsymbol{\nu}_{\mathrm{u}}$ and $\boldsymbol{\nu}_{\mathrm{s}}$, respectively, represent the unsteady and steady components of the flow velocity in dimensions consistent with $\boldsymbol{\nu}$:

$$\boldsymbol{\nu} = \begin{pmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{pmatrix}, \quad \boldsymbol{\nu}_{\mathrm{u}} = \begin{pmatrix} \boldsymbol{v}_{\mathrm{u}} \\ \boldsymbol{0} \end{pmatrix} \quad \text{and} \quad \boldsymbol{\nu}_{\mathrm{s}} = \begin{pmatrix} \boldsymbol{v}_{\mathrm{s}} \\ \boldsymbol{0} \end{pmatrix}.$$

The generalized velocity relative to the flow is

$$\boldsymbol{\nu}_{\mathrm{r}} = \boldsymbol{\nu} - \boldsymbol{\nu}_{\mathrm{u}} - \boldsymbol{\nu}_{\mathrm{s}} = \begin{pmatrix} \boldsymbol{v}_{\mathrm{r}} \\ \boldsymbol{\omega} \end{pmatrix}$$

where $\boldsymbol{v}_{\mathrm{r}} = \boldsymbol{v} - \boldsymbol{v}_{\mathrm{u}} - \boldsymbol{v}_{\mathrm{s}}$ is the flow-relative translational velocity of the vehicle, written in the body frame.

Referring to [5] and [8], the "full" dynamic equations of AUV in currents can be derived, in terms of flow-relative velocity

$$\begin{aligned}
(\mathbb{M}_{\mathrm{f}} + \mathbb{M})\,\dot{\boldsymbol{\nu}}_{\mathrm{r}} = &-\begin{pmatrix} \hat{\boldsymbol{\omega}} & \boldsymbol{0} \\ \hat{\boldsymbol{v}}_{\mathrm{r}} & \hat{\boldsymbol{\omega}} \end{pmatrix}(\mathbb{M}_{\mathrm{f}} + \mathbb{M})\,\boldsymbol{\nu}_{\mathrm{r}} + \begin{pmatrix} \boldsymbol{f} \\ \boldsymbol{m} \end{pmatrix} \\
&- \begin{pmatrix} \hat{\boldsymbol{\omega}} & \boldsymbol{0} \\ \hat{\boldsymbol{v}}_{\mathrm{r}} + \hat{\boldsymbol{v}}_{\mathrm{u}} + \hat{\boldsymbol{v}}_{\mathrm{s}} & \hat{\boldsymbol{\omega}} \end{pmatrix}(\mathbb{M} - \bar{\mathbb{M}})\begin{pmatrix} \boldsymbol{v}_{\mathrm{u}} + \boldsymbol{v}_{\mathrm{s}} \\ \boldsymbol{0} \end{pmatrix} \\
&- \begin{pmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \hat{\boldsymbol{v}}_{\mathrm{u}} + \hat{\boldsymbol{v}}_{\mathrm{s}} & \boldsymbol{0} \end{pmatrix}(\mathbb{M} - \bar{\mathbb{M}})\,\boldsymbol{\nu}_{\mathrm{r}} \\
&- (\mathbb{M} - \bar{\mathbb{M}})\begin{pmatrix} \begin{pmatrix} (\boldsymbol{v}_{\mathrm{u}} + \boldsymbol{v}_{\mathrm{s}}) \times \boldsymbol{\omega} + \frac{\partial}{\partial t}\boldsymbol{v}_{\mathrm{u}} \\ +\boldsymbol{\Phi}^T(\boldsymbol{v}_{\mathrm{r}} + \boldsymbol{v}_{\mathrm{u}} + \boldsymbol{v}_{\mathrm{s}}) \end{pmatrix} \\ \boldsymbol{0} \end{pmatrix} \\
&- \begin{pmatrix} \boldsymbol{\Phi} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{pmatrix}(\mathbb{M}_{\mathrm{f}} + \bar{\mathbb{M}})\,\boldsymbol{\nu}_{\mathrm{r}}
\end{aligned} \tag{3}$$

where

$$\boldsymbol{\Phi} = \boldsymbol{R}^T\left(\frac{\partial \boldsymbol{V}_{\mathrm{s}}}{\partial \boldsymbol{X}}\right)^T\boldsymbol{R}. \tag{4}$$

while $\mathbb{M}$ and $\mathbb{M}_{\mathrm{f}}$ are the 6×6 "generalized inertia" matrix and "generalized added inertia" matrix for the rigid vehicle, respectively; $\bar{\mathbb{M}}$ denotes a mass matrix that accounts for the kinetic energy of the fluid that is replaced by the vehicle.

Note that aside from the first line of equations (3), the other terms describe the currents effects on AUV dynamics. While the external forces and moments acting on the body are those due to

- gravity and buoyancy ($\boldsymbol{f}_{\mathrm{g/b}}$ and $\boldsymbol{m}_{\mathrm{g/b}}$)
- viscous effects ($\boldsymbol{f}_{\mathrm{v}}$ and $\boldsymbol{m}_{\mathrm{v}}$), and
- control forces and moments ($\boldsymbol{f}_{\mathrm{ctrl}}$ and $\boldsymbol{m}_{\mathrm{ctrl}}$)

Control forces and moments are typically generated using external effectors, e.g., propeller's rotation and control planes' deflections. The complete external force and moment are

$$\begin{aligned} \boldsymbol{f} &= \boldsymbol{f}_{\mathrm{g/b}} + \boldsymbol{f}_{\mathrm{v}} + \boldsymbol{f}_{\mathrm{ctrl}} \\ \boldsymbol{m} &= \boldsymbol{m}_{\mathrm{g/b}} + \boldsymbol{m}_{\mathrm{v}} + \boldsymbol{m}_{\mathrm{ctrl}}. \end{aligned}$$

Combining the kinematic equations (1), (2), with the dynamic equations (3), one obtains a complete dynamic model for an AUV in an unsteady, nonuniform flow field. The AUV model used for the following simulations is described in [9]. The AUV hull is a prolate spheroid 2 meters long with a fineness ratio of 10:1. The four identical tail fins, arranged in a cruciform configuration, have an aspect ratio of 3; the tip-to-tip span for each pair of fins is 50 centimeters. The vessel is trimmed to be neutrally buoyant and the propeller is fixed such that the nominal speed is $\tilde{u} > 0$. Vehicle attitude is regulated by rudder and elevator through proportional-derivative feedback.

III. Observer Design and Numerical Simulation

In this section, we will design a nonlinear observer based on AUV dynamic model for current velocity estimation. Motivated by the design method of high-gain observer, in our case, we take the current disturbances as the uncertainties of the vehicle dynamic system and estimate vehicle's relative velocity firstly; current velocity is further calculated in an indirect way.

A. *Observer Design*

We actually use rotational, kinematic equations (2) and dynamic equations (3) to set up the nonlinear observer. The system dynamics can be simply described as this

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{d}) \quad (5)$$

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} \quad (6)$$

where $\boldsymbol{x} = [\phi,\ \theta,\ \psi,\ u_r,\ v_r,\ w_r,\ p,\ q,\ r]^T$ is the state vector; $\boldsymbol{u} = [n, \delta_r, \delta_e]^T$ is the control input, n denotes the propeller's rotation and δ_r, δ_e denote the respective deflections of rudder and elevator; $\boldsymbol{d}$ denotes the current disturbances; $\boldsymbol{y}$ is the output vector; $\boldsymbol{C}$ is the measurement matrix. The function $\boldsymbol{f}$ is locally Lipschitz in $(\boldsymbol{x}, \boldsymbol{u})$ and continuous in $\boldsymbol{d}$.

The observer design is based on the following assumptions and properties:

- The current is time-varying, nonuniform but upper bounded, i.e. $\exists \mathbb{V}_{\mathrm{f}} \in \mathbb{R}_+$ such that $\mathbb{V}_{\mathrm{f}} = \sup_{\boldsymbol{X},t} \|\boldsymbol{V}_{\mathrm{f}}(\boldsymbol{X}, t)\|$.
- The the vehicle's absolute velocity vector and Euler angles are measurable.

Usually, Euler angles can be sensed by electronic compass or inertial navigation system (INS); vehicle's absolute velocity can be directly measured by Doppler velocity logger (DVL) or indirectly obtained by position differentiating. Someone may say vehicle's relative velocity can also be measured by DVL, while there exists a blind zone that certain minimum number of cell layer is required for DVL to get water profiling, which causes vehicle's velocity relative to the water around cannot be measured accurately. Here, we consider that vehicle's relative velocity is not measurable.

To implement the feedback control using only measurements of the output $\boldsymbol{y}$, we design the observer

$$\dot{\hat{\boldsymbol{x}}} = \hat{\boldsymbol{f}}(\hat{\boldsymbol{x}}, \boldsymbol{u}) + \boldsymbol{G}(\boldsymbol{y} - \boldsymbol{C}\hat{\boldsymbol{x}}) \quad (7)$$

where $\hat{\boldsymbol{f}}(\hat{\boldsymbol{x}}, \boldsymbol{u})$ is a model of $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{d})$; here, we take the current disturbances as the uncertainties of the vehicle dynamics. Since $\boldsymbol{f}$ is a known function of $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$, we can take $\hat{\boldsymbol{f}} = \boldsymbol{f}$. The estimation error

$$\tilde{\boldsymbol{x}} = \boldsymbol{x} - \hat{\boldsymbol{x}}$$

satisfies the equation

$$\dot{\tilde{\boldsymbol{x}}} = -\boldsymbol{G}\boldsymbol{C}\tilde{\boldsymbol{x}} + \boldsymbol{\delta}(\boldsymbol{x}, \tilde{\boldsymbol{x}}, \boldsymbol{d}) \quad (8)$$

where

$$\boldsymbol{\delta}(\boldsymbol{x}, \tilde{\boldsymbol{x}}, \boldsymbol{d}) = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{d}) - \hat{\boldsymbol{f}}(\hat{\boldsymbol{x}}, \boldsymbol{u}).$$

In the absence of $\boldsymbol{\delta}$, asymptotic error convergence is achieved when the matrix $-\boldsymbol{G}$ is Hurwitz. In the presence of $\boldsymbol{\delta}$, we should carefully design the observer gain matrix $\boldsymbol{G}$ with the additional goal of rejecting the effect of $\boldsymbol{\delta}$ on $\tilde{\boldsymbol{x}}$ and making the observer robust to uncertainties in modeling the nonlinear functions.

In our case, the observer gain matrix $\boldsymbol{G}$ is preliminarily optimized by pole placement, which assigns the eigen values at the chosen poles, such that

$$\operatorname*{limit}_{\mathrm{t}\to\infty} \tilde{\boldsymbol{x}}(\mathrm{t}) = 0.$$

Besides, Euler angles are used as the measurements for our observer design, while the measurement matrix can be defined as this

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Therefore, with the nonlinear observer, vehicle's relative velocity $\boldsymbol{v}_{\mathrm{r}} = [u_{\mathrm{r}},\ v_{\mathrm{r}},\ w_{\mathrm{r}}]^T$ can be firstly estimated; then the current velocity can be further calculated by subtracting vehicle's relative velocity $\boldsymbol{v}_{\mathrm{r}}$ from vehicle's absolute velocity $\boldsymbol{v}$ as follows:

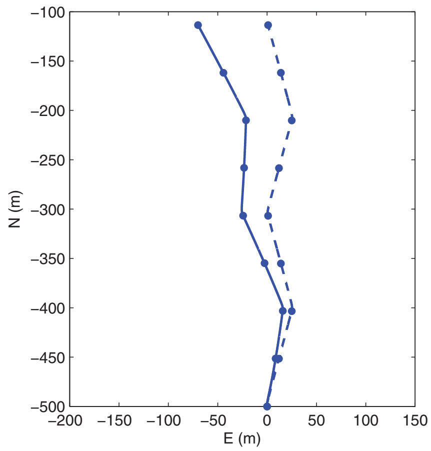
$$\boldsymbol{v}_{\mathrm{f}} = \boldsymbol{v} - \boldsymbol{v}_{\mathrm{r}}$$

where $\boldsymbol{v}_{\mathrm{f}}$ is current velocity expressed in the body frame, multiplied by the rotation matrix $\boldsymbol{R}$, current velocity expressed in the inertial frame can be obtained

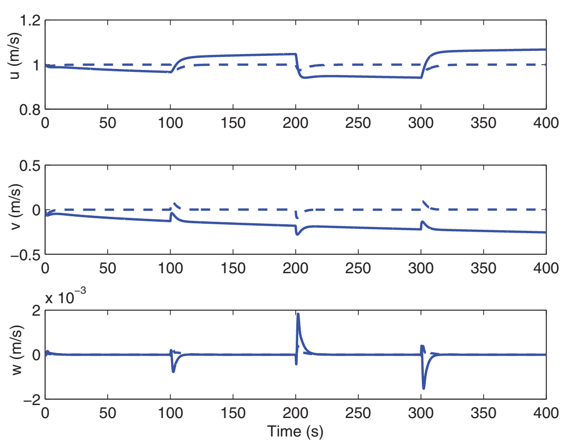
$$\boldsymbol{V}_{\mathrm{f}} = \boldsymbol{R}\boldsymbol{v}_{\mathrm{f}}$$

B. Numerical Simulation

The proposed nonlinear observer is validated by numerical simulation here. Comparing with [6], we assume that the vehicle runs in a nonuniform flow, which is defined as $\boldsymbol{V}_{\mathrm{f}}(\boldsymbol{X}) = [0, -0.3\sqrt{(500+\xi)/500}, 0]^T$ m/s for $\xi > -500$ m. The vehicle executes a sawtooth pattern from south to north in the horizontal plane at a nominal speed $V = 0.77$ m/s. The vehicle's reference heading switches from $\psi = 15°$ to $\psi = -15°$ with a time interval of 100 s.



(a) Horizontal pathes in inertial frame.



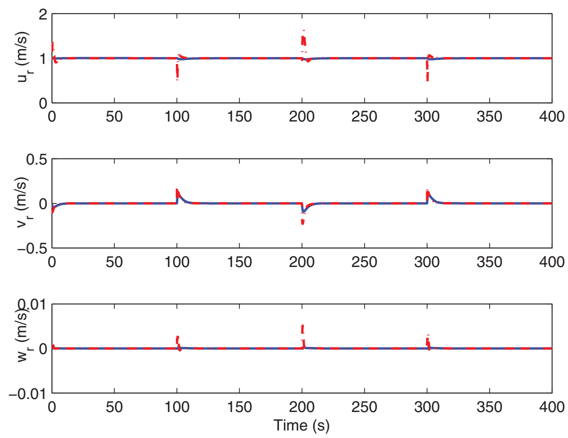
(b) Absolute velocities in body frame.

Fig. 2. Comparison of AUV paths and absolute velocities with (solid blue curve) and without (dashed blue curve) current disturbance. Blue dots indicate 50 s intervals.

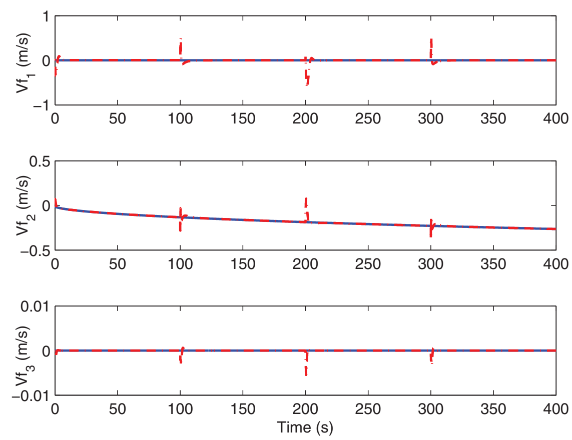
AUV starts form the point $\boldsymbol{X} = [-500, 0, 0]^T$ with initial attitude $\psi = \phi = \theta = 0$. Figure 2(a) shows the horizontal plane paths that result from two scenarios when the vehicle runs in the water with current disturbance or not, respectively. Both cases are AUV motion prediction using the full dynamic equations derived in Section II. Below is a summary of the simulation cases.

- AUV motion simulated with current disturbance. (Solid blue curve)
- AUV motion simulated without current disturbance. (Dashed blue curve)

There is a noticeable discrepancy between the two paths in Figure 2(a), which is mainly due to the disturbed vehicle velocity caused by the current; see Figure 2(b). If the current velocity equals to 0, the solid and dashed curves in Figures 2(b) will coincide.



(a) Relative velocities in body frame.



(b) Current velocities in inertial frame.

Fig. 3. Estimated results of nonlinear observer. The solid blue curve represents the actual value and the dash red curve is the estimated one.

Using the proposed nonlinear observer, with the measured Euler angles, the vehicle's relative velocity can be observed, as is shown in Figure 3(a). Furthermore, the current velocity can be estimated indirectly by subtracting the observed relative velocity from the measured absolute velocity; see Figure 3(b). It can be found that there is good agreement between the true and estimated current velocities, except some transient burrs during heading control.

IV. Conclusions

This paper presents a nonlinear observer for current velocity estimation based on AUV dynamic model in currents. Motivated by the design method of high-gain observer, we take the current disturbances as the uncertainties of the vehicle dynamic system; the observer gain is optimized by pole placement to make the observer robust to uncertainties in modeling the nonlinear functions. In our case, vehicle's relative velocity is firstly estimated; current velocity is further calculated by subtracting vehicle relative velocity from vehicle absolute velocity. Numerical simulation demonstrates the performance of the proposed nonlinear observer, showing good agreement between the true and estimated current velocities. Ongoing efforts will focus on the rigorous theoretical derivation of system observability, determination of the disturbance bound and observer gain optimization.

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